# Beam Propagation Analysis of a Tapered Proton-Exchanged Lithium Niobate Optical Waveguide

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***Abstract*—A simulation of tapered lithium niobate optical waveguide fabricated by the proton-exchanged method is presented. The Fresnel equation with an initial input field distribution is solved numerically using the semivectorial-polarized finite difference method and the Runge–Kutta method. The calculated and experimental results are in good agreement.**

## I. Introduction

On the basis of the second harmonic generation, compact solid-state laser diodes have been widely used as an efficient light source for optical storage and bio-medical applications. In order to improve the efficiency of second harmonic generation, it is important to increase the coupling from the laser diode to the waveguide in the endfire configuration. Recently, the use of a tapered proton-exchanged (PE) waveguide in lithium niobate to reduce the coupling loss has been reported [1]. However, theoretical analysis of the device performance was not given. In this work, the beam propagation method with a semivectorial approximation [2], [3] is used to analyze the tapered waveguide. Good agreements between the calculated and measured coupling losses are obtained.

The numerical methods commonly used for solving the Fresnel equation are the fast Fourier transform method (FFTM) [4] and the finite difference method (FDM). However, it was reported that the equation discretized by FDM is more efficient and stable than that by FFTM [5], [6]. Usually, FDM starts with a discritization of the Fresnel equation, including both the transverse and the propagation components, by the central difference scheme [5]–[7]. However, the Crank–Nicolson scheme for the propagation component is implicit, which takes a lot of computing time, especially when the full-wave analysis is considered. To save the computing time, the implicit Crank–Nicolson is replaced by the explicit Runge–Kutta fourth order formulas [7]. It is well known that the Runge–Kutta method is derived mainly for ordinary differential equations, however, in this work, it is shown that this method is also good for partial differential equations such as the Fresnel equation.

## II. Theory

Consider a rectangular waveguide of width $W$, height $H$, and refractive index $n_g$ embedded in a substrate of refractive index $n_s$. as shown in Fig. 1. With the assumption that the guided optical wave is slowly varying and paraxially propagating, the Helmholtz equation can be reduced to the Fresnel equation as given by

$$2jk_0 n_s \frac{\partial E}{\partial z} = f + g \tag{1}$$

where

$$f = \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2}; \tag{2}$$

$$g = k_0^2 \left[ n^2(x,y,z) - n_s^2 \right] E, \tag{3}$$

$k_0$ is the free space wave number, $E$ is the propagating field, and $n(x, y, z)$ is the index distribution. As the proton-exchanged waveguides can support only the extraordinary-polarized (i.e., $y$-polarized in this work) waves [8], The semivectorial–polarized finite difference approximation for the second order derivatives of $E$ can be written as follows

$$\frac{\partial^2 E_{i,j}}{\partial x^2} = \frac{E_{i+1,j} - 2E_{i,j} + E_{i-1,j}}{\Delta x^2} \tag{4}$$

$$\begin{aligned}\frac{\partial^2 E_{i,j}}{\partial y^2} =& \frac{2n_{i,j+1}^2 E_{i,j+1}}{\Delta y^2 (n_{i,j+1}^2 + n_{i,j}^2)} + \frac{2n_{i,j-1}^2 E_{i,j-1}}{\Delta y^2 (n_{i,j}^2 + n_{i,j-1}^2)} \\ &- \left[ \frac{1}{(n_{i,j+1}^2 + n_{i,j}^2)} + \frac{1}{(n_{i,j}^2 + n_{i,j-1}^2)} \right] \frac{2n_{i,j}^2 E_{i,j}}{\Delta y^2}\end{aligned} \tag{5}$$

Note that (4) and (5) have taken into account the large index discontinuities along the $x$ and $y$ directions of the waveguide cross section. For the quasi-TE modes, similar results can be obtained by interchanging $x$ and $y$. With the above discretization, (1) becomes a system of first order differential equations. Obviously, there are a lot of numerical methods can be used to obtain a solution. For simplicity, we use the well known fourth order Runge–Kutta method. The reason can be understood by considering the right hand side of (1). When $f$ is zero, (1) becomes an ordinary differential equation. Usually, the Runge–Kutta method is used to obtain a numerical solution. However, when $g$ is zero, (1) is reduced to a parabolic partial differential equation. The Crank–Nicolson scheme is used instead. In the case of the Fresnel equation, both $f$ and $g$ in (1) are nonzeros, neither of the above methods can be applied. However, the waveguiding characteristic are described by the function $g$, which is the dominant part of the equation. The differential terms involving $f$ are only served as the

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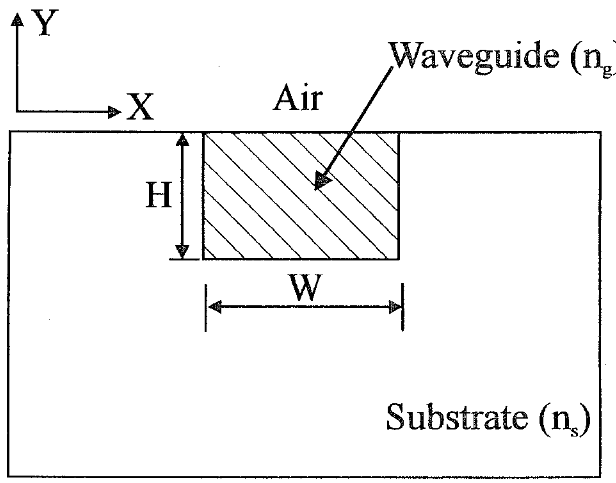


Fig. 1. Typical embedded waveguide structure.

"couplings" between different ordinary differential equations when the $x$ and $y$ coordinates of the Fresnel equation are discretized. Hence, it is reasonable to discretize (1) as a system of ordinary differential equations.

The stability and accuracy of our program were tested using some typical values of the previous experiment [1] as given by $n_s = 2.2$, $n_g = 2.32$, $W = 1.8$ $\mu$m, and $H = 0.36$ $\mu$m. The grid sizes are $\Delta x = 0.1$ $\mu$m, $\Delta y = 0.05$ $\mu$m, and $\Delta z = 0.025$ $\mu$m. The power conservation of the fundamental mode propagating a distance of 1 cm is less than 0.1%. Also, the propagation constant of the fundamental mode is calculated to be 16.7874 by our method and 16.7812 by the finite element method. These two tests show that the accuracy and stability of our method are good enough for the beam propagation analysis.

## III. Simulation

Consider a tapered proton-exchanged lithium niobate waveguide as shown in Fig. 2. The incident beam emitted from a laser diode has a width of 4 $\mu$m and a height of 1.6 $\mu$m. The total length of this waveguide is 20 mm. The waveguide can be divided into an endfire coupling section, a tapered section, and a single-mode waveguide section. The dimensions of the coupling section are 4 $\mu$m $\times 1.5$ $\mu$m $\times 1.1$ mm (width $\times$ depth $\times$ length), and the single-mode waveguiding section are 1.8$\mu$m $\times 0.36\mu$m $\times 18$ mm. The tapered section consists of a width taper and a depth taper. The width taper narrows the waveguide from 4$\mu$m to 1.8$\mu$m over a length of 800$\mu$m. The depth taper takes the waveguide from 1.5$\mu$m up to 0.36$\mu$m over a length of 60$\mu$m. In our simulation, for simplicity, only the power transfer occurs in the tapered section is considered. The propagating fields in the tapered section at $z = 200$, 400, 800, and 950 $\mu$m are shown in Fig. 3. As can be seen from the figure, the propagating field does change its size in width first, then depth, and finally, fit into the single-mode waveguide. The coupling loss between the laser diode and the single mode waveguide is calculated to be 1.39 dB with the taper and 2.29 dB without. Similarly, the experimental results [1] are 1.3dB and 2.3 dB. Obviously, the theoretical results agree quite well with those obtained experimentally.

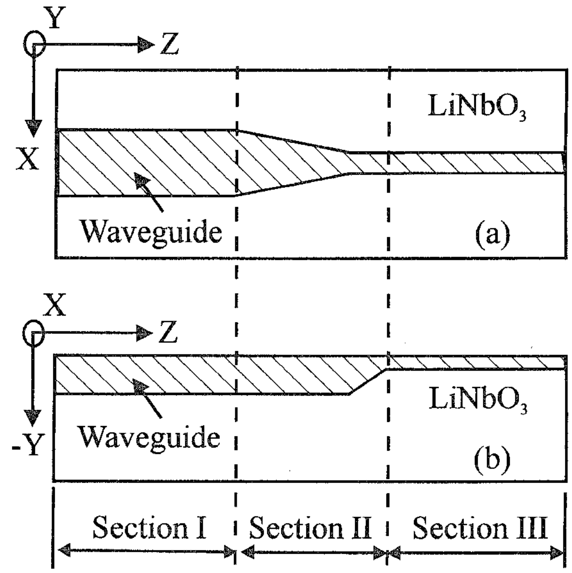


Fig. 2. Illustrations of the tapered proton-exchanged lithium niobate waveguide. (a) top view and (b) side view. The waveguide is divided into an endfire coupling section (Section I), a tapered section (Section II), and a single-mode waveguide section (Section III).

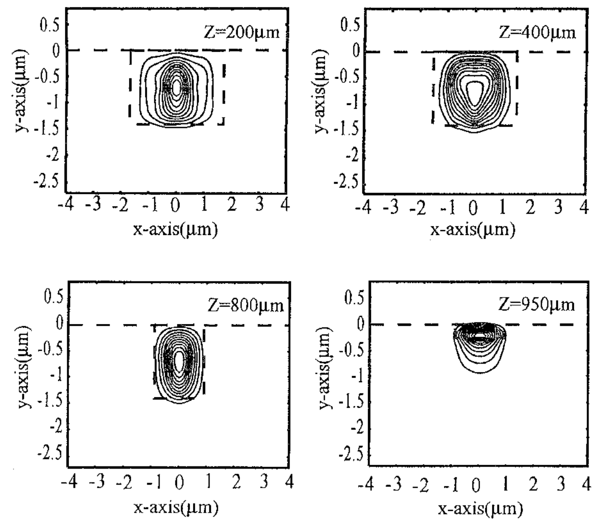


Fig. 3. Contour plots of optical power distributions within the tapered section at different positions: (a)200 $\mu$m, (b) 400 $\mu$m, (c) 800 $\mu$m, and (d) 950 $\mu$m. The guiding region is shown by dashed lines and contour levels are at 10% intervals of the maximum field.

## IV. Conclusion

In conclusion, the tapered proton-exchange lithium niobate waveguide for the reduction of the edge coupling loss is successively simulated. The calculated results show that the combination of the Runge–Kutta and the semivectorial–polarized finite difference method is a simple and effective way for beam propagation analysis. Further application on the beam propagation analysis of other waveguide devices is of great interest in the future. Moreover, Runge–Kutta method is not the only way to find the numerical solution of the system of first order differential equation discretized from the Fresnel equation, the other methods, such as the predictor corrector method, can also be used. Detailed comparison of the results will be discussed in the future.

## REFERENCES


[1] K. Mizuuchi, K. Yamamoto, and T. Taniuchi, "High-efficiency coupling of a laser diodes in tapered proton-exchanged waveguides," *Electron. Lett.*, vol. 26, no. 24, pp. 1992–1994, 1992.

[2] M. S. Stern, "Semivectorial polarized finite difference method for optical waveguides with arbitrary index profiles," *Inst. Elect. Eng. Proc.*, vol. 135, pt. J, no. 1, pp. 56–63, 1988.

[3] P. L. Liu and B. J. Li, "Semivectorial beam-propagation method for analyzing polarized modes of rib waveguides," *IEEE J. Quantum Electron.*, vol. 28, pp. 778-782, 1992.

[4] M. D. Feit and J. A. Fleck, Jr., "Light propagation in graded-index optical fibers," *Appl. Phys. Lett.*, vol. 17, no. 24, pp. 3990–3998, 1978.

[5] Y. Chung and N. Dagli, "An assessment of finite difference beam propagation methods," *IEEE J. Quantum Electron.*, vol. 26, pp. 1335–1339, 1990.

[6] R. Scarmozzino and R. M. Osgood, Jr., "Comparison of finite-difference and Fourier-transform solutions of the parabolic wave equation with emphasize on integrated optics application," *J. Opt. Soc. Amer. A*, vol. 8, no. 5, pp. 724–731, 1991.

[7] W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes: The Art of Scientific Computing* London, United Kingdom: Cambridge University Press, pp. 550–554.

[8] J. L. Jackel, C. E. Rice, and J. J. Veselka, "Proton exchange for high-index waveguides in $LiNbO_3$," *Appl. Phys. Lett.*, vol. 41, no. 7, pp. 607–608, 1982.